

Section 10-4

Perimeter and area of similar figures

Theorem 10-7 Perimeters and Areas of Similar Figures

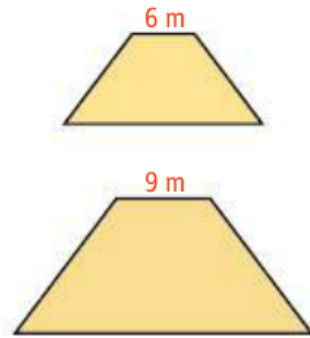
If the scale factor of two similar figures is $\frac{a}{b}$, then

(1) the ratio of their perimeters is $\frac{a}{b}$ and

(2) the ratio of their areas is $\frac{a^2}{b^2}$.

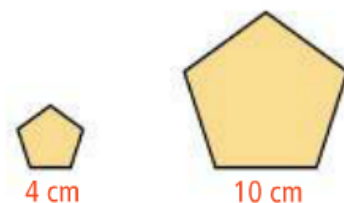
Example

The ratio of the corresponding sides is 2:3. Without calculating the area we know it must be $2^2:3^2$ or 4:9.



Example

If the smaller pentagon has an area of 27.5 cm^2 , what is the area of the larger pentagon?

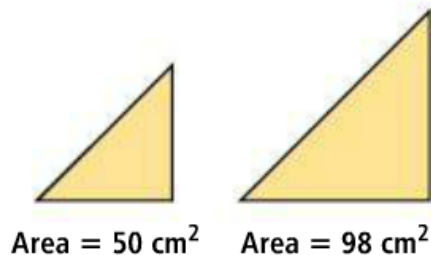


Since the ratio of corresponding sides is 4:10 or 2:5 we know the ratio of areas must be 4:25, so:

$$\frac{4}{25} = \frac{27.5}{x} \quad x = 171.875$$

Example

We can also go from area to perimeter. Given the areas to the right, what is the ratio of the perimeters?



$$\frac{50}{98} = \frac{a^2}{b^2}$$

$$\frac{25}{49} = \frac{a^2}{b^2}$$

$$\frac{5}{7} = \frac{a}{b}$$

Area and trigonometry

$$\text{SOH} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{CAH} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

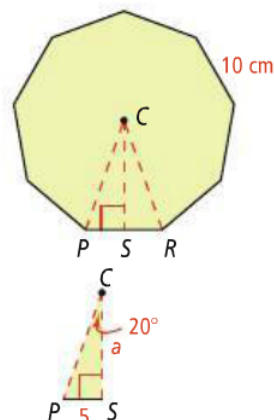
$$\text{TOA} = \frac{\text{opposite}}{\text{adjacent}}$$

What is the area of a regular nonagon with 10cm sides?

Draw a regular nonagon with center C . Draw \overline{CP} and \overline{CR} to form isosceles $\triangle PCR$. The measure of central $\angle PCR$ is $\frac{360}{9}$, or 40° . The perimeter is $9 \cdot 10$, or 90 cm. Draw the apothem \overline{CS} .

$$m\angle PCS = \frac{1}{2} m\angle PCR = 20^\circ$$

$$PS = \frac{1}{2} PR = 5\text{cm}$$



What is the area of a regular nonagon with 10cm sides?

Let a represent CS . Find a and substitute into the area formula.

$$\tan 20^\circ = \frac{5}{a} \quad \text{Use the tangent ratio.}$$

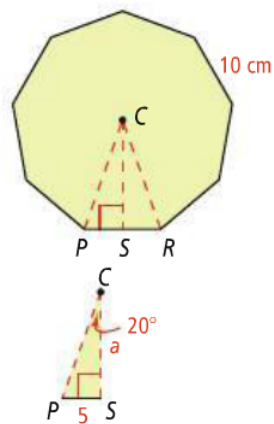
$$a = \frac{5}{\tan 20^\circ} \quad \text{Solve for } a.$$

$$A = \frac{1}{2} ap$$

$$= \frac{1}{2} \cdot \frac{5}{\tan 20^\circ} \cdot 90 \quad \text{Substitute } \frac{5}{\tan 20^\circ} \text{ for } a \text{ and } 90 \text{ for } p.$$

$$\approx 618.1824194 \quad \text{Use a calculator.}$$

The area of the regular nonagon is about 618 cm^2 .



What is the area of a regular octagon with a radius of 16.2 cm?

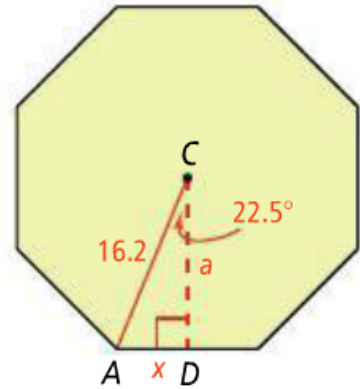
Let a represent the apothem. Use the cosine ratio to find a .

The measure of a central angle of the octagon is $\frac{360}{8}$, or 45.

So $m\angle C = \frac{1}{2}(45) = 22.5$.

$$\cos 22.5^\circ = \frac{a}{16.2} \quad \text{Use the cosine ratio.}$$

$$16.2(\cos 22.5^\circ) = a \quad \text{Multiply each side by 16.2.}$$

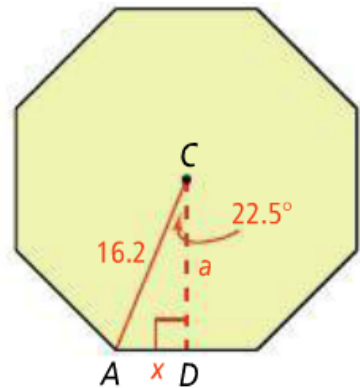


What is the area of a regular octagon with a radius of 16.2 cm?

Let x represent AD . Use the sine ratio to find x .

$$\sin 22.5^\circ = \frac{x}{16.2} \quad \text{Use the sine ratio.}$$

$$16.2(\sin 22.5^\circ) = x \quad \text{Multiply each side by 16.2.}$$



What is the area of a regular octagon with a radius of 16.2 cm?

Find the perimeter of the octagon.

$$p = 8 \cdot \text{length of a side}$$

$$= 8 \cdot 2x$$

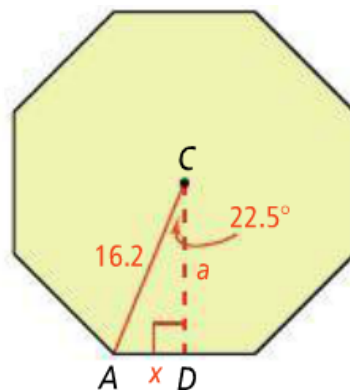
The length of each side is $2x$.

$$= 8 \cdot 2 \cdot 16.2(\sin 22.5^\circ)$$

Substitute for x .

$$= 259.2(\sin 22.5^\circ)$$

Simplify.



What is the area of a regular octagon with a radius of 16.2 cm?

Substitute into the area formula.

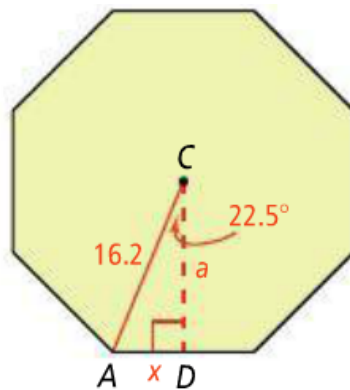
$$A = \frac{1}{2}ap$$

$$= \frac{1}{2} \cdot 16.2(\cos 22.5^\circ) \cdot 259.2(\sin 22.5^\circ)$$

Substitute for a and p .

$$\approx 742.2924146$$

Use a calculator.



In class task!

Find the area of the following regular polygons

octagon with side length 6 cm

pentagon with radius 3 ft

dodecagon with radius 20 cm

18-gon with perimeter 72 mm

In class task!

Find the area of the following regular polygons

octagon with side length 6 cm = 173.8 cm^2

pentagon with radius 3 ft = 21.4 ft^2

dodecagon with radius 20 cm = 1200 cm^2

18-gon with perimeter 72 mm = 408.3 mm^2

Have a great weekend!

No homework!

(MathXL due Monday!)